Introduction to atomic clocks and statistics for time and frequency

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CLONETS

CLOCK NETWORK SERVICES

Overview

- Common atomic clocks
- Construction and use of Allan Deviation (ADEV), and similar statistics
- Clock predictability
- Conclusions

Atomic Clocks

- Timekeeping is based on atomic transition frequencies.

- Primary Frequency Standards

  Caesium Fountain Primary Standards, Optical clocks

- High quality commercial atomic clocks

  Active Hydrogen Masers, Commercial Caesium Clocks,

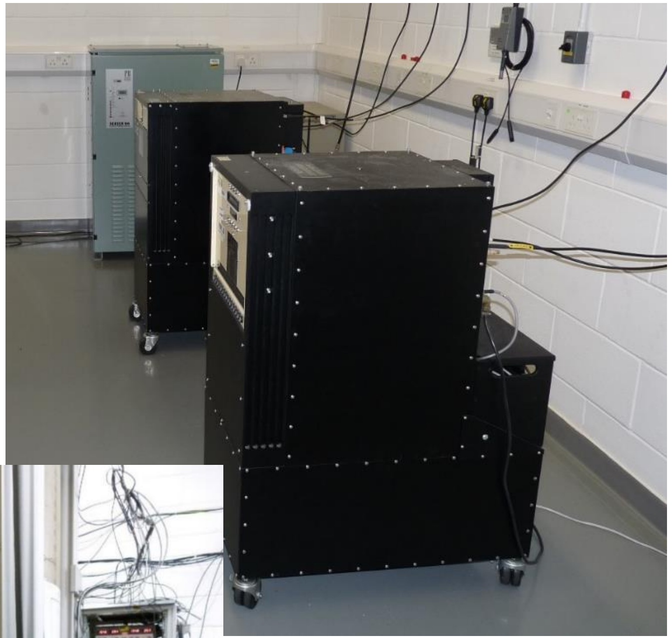
  Space Rubidium Atomic Frequency Standards

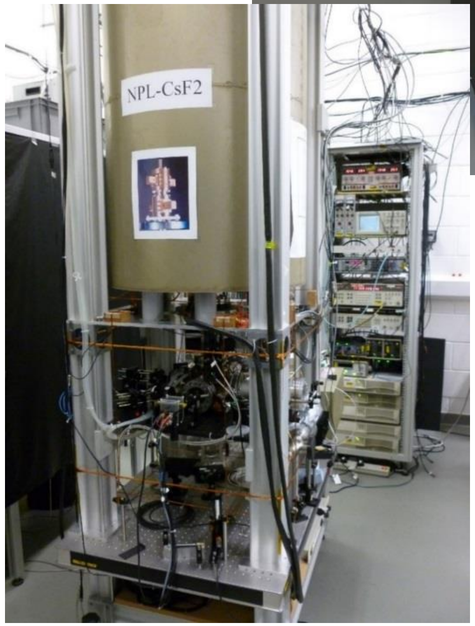
- More widely used atomic clocks

  Rubidium Frequency Standards

- Miniaturised Atomic clocks

  CSACs





Deterministic and Noise Processes

Deterministic Properties

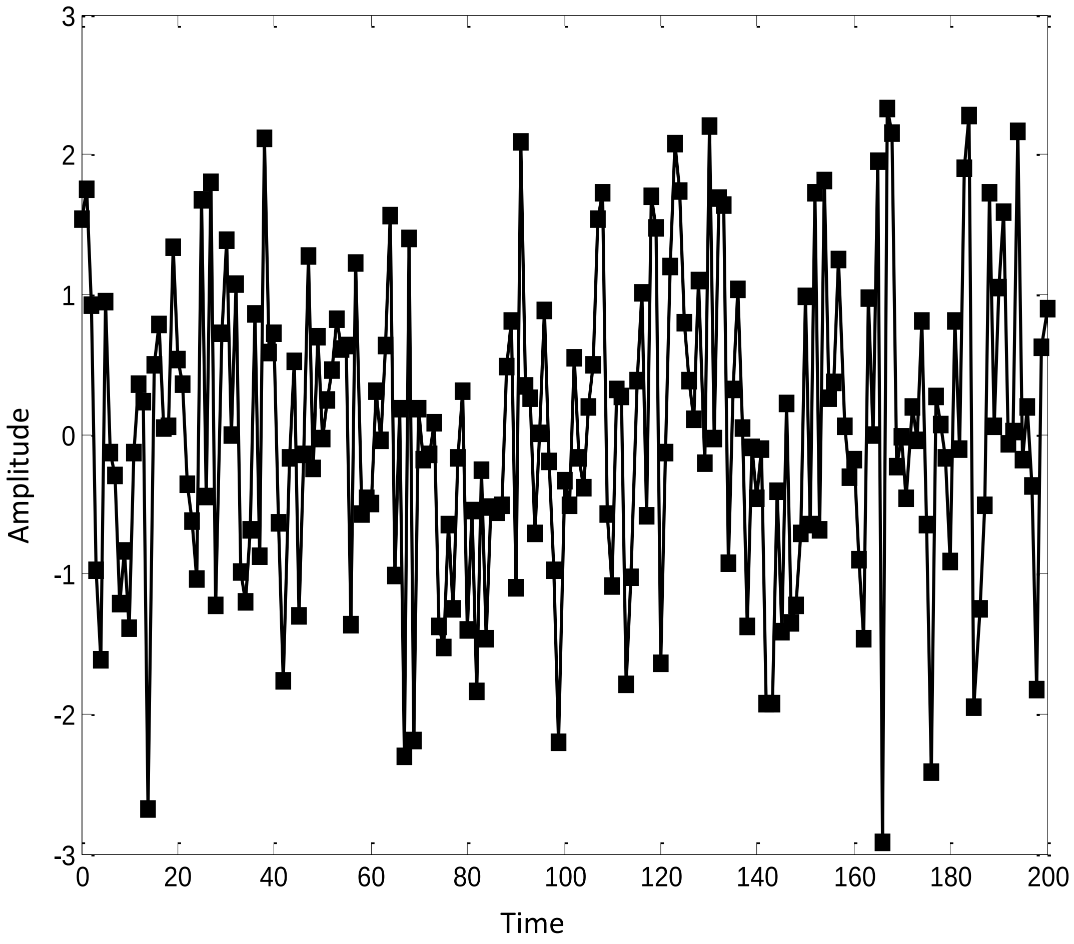
Time offset,
Normalised frequency Offset
Linear Frequency Drift (Active Hydrogen Masers and Rubidium Clocks)

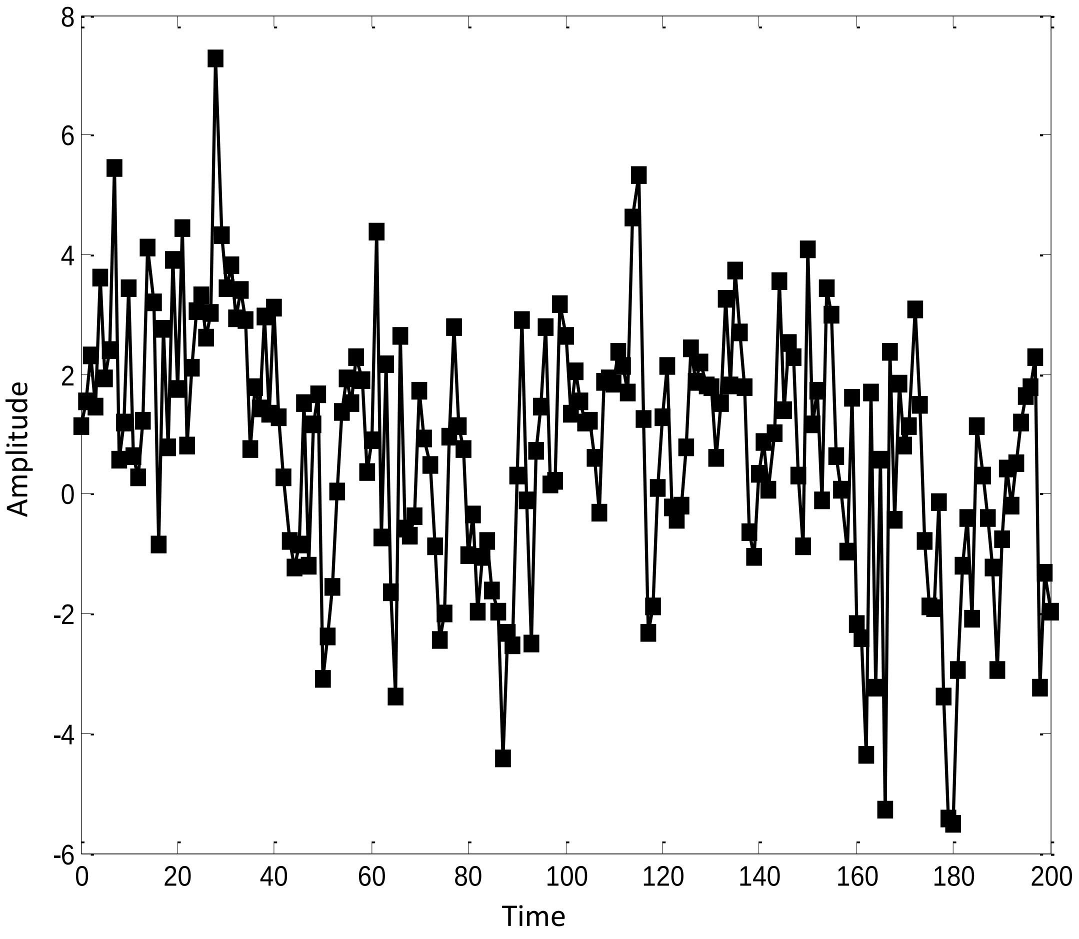
Stochastic (Random) Noise Properties

Power Law Noise Processes
Other noise processes, often caused by measurement noise and time transfer systems.
Periodic effects e.g. diurnal changes

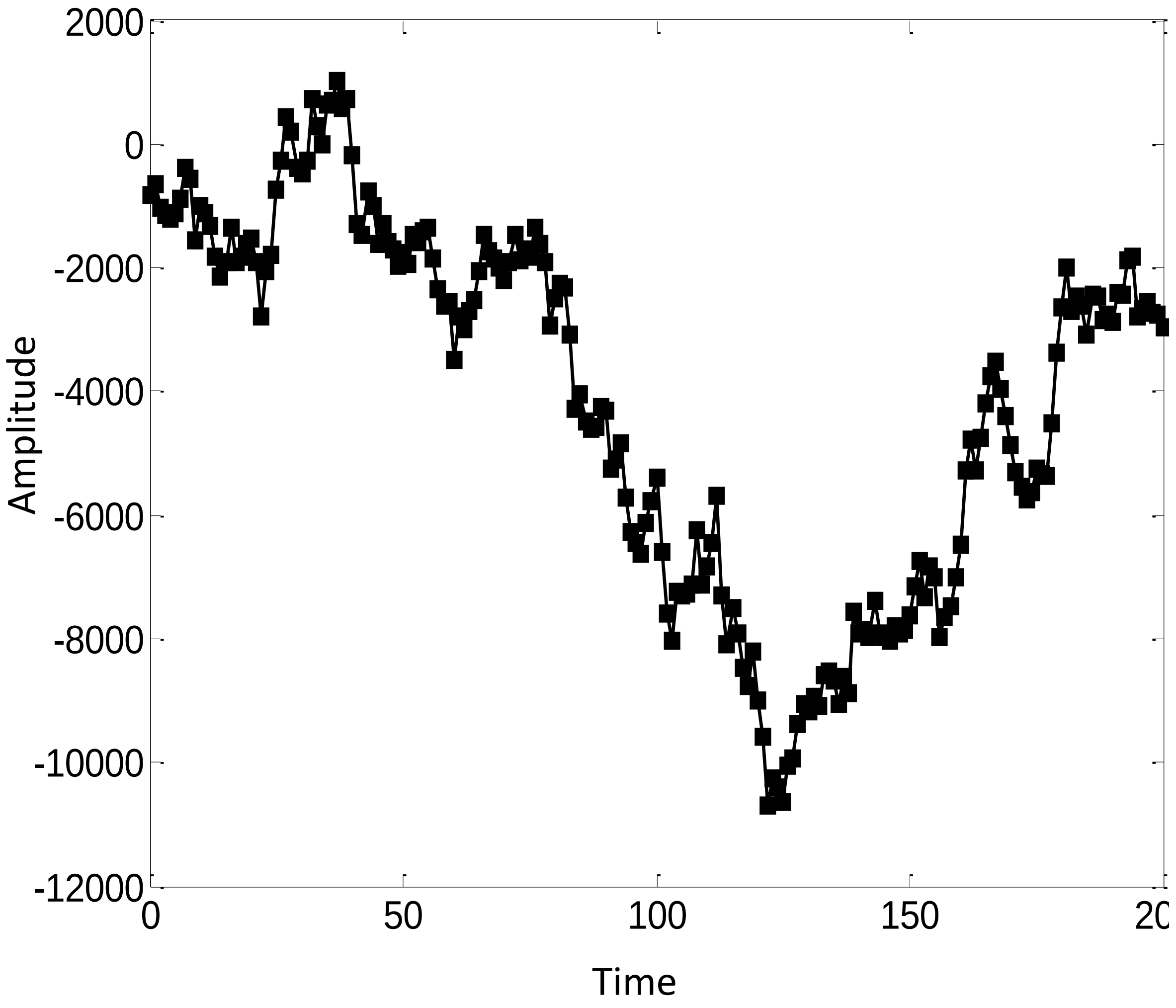
Noise type	Power Spectral Density of fractional frequency measurements $S_y(f)$
White Phase Modulation (WPM)	$h_2 f^2$
Flicker Phase Modulation (FPM)	$h_1 f$
White Frequency Modulation (WFM)	h_0
Flicker Frequency Modulation (FFM)	$h_{-1} f^{-1}$
Random Walk Frequency Modulation (RWFM)	$h_{-2} f^{-2}$

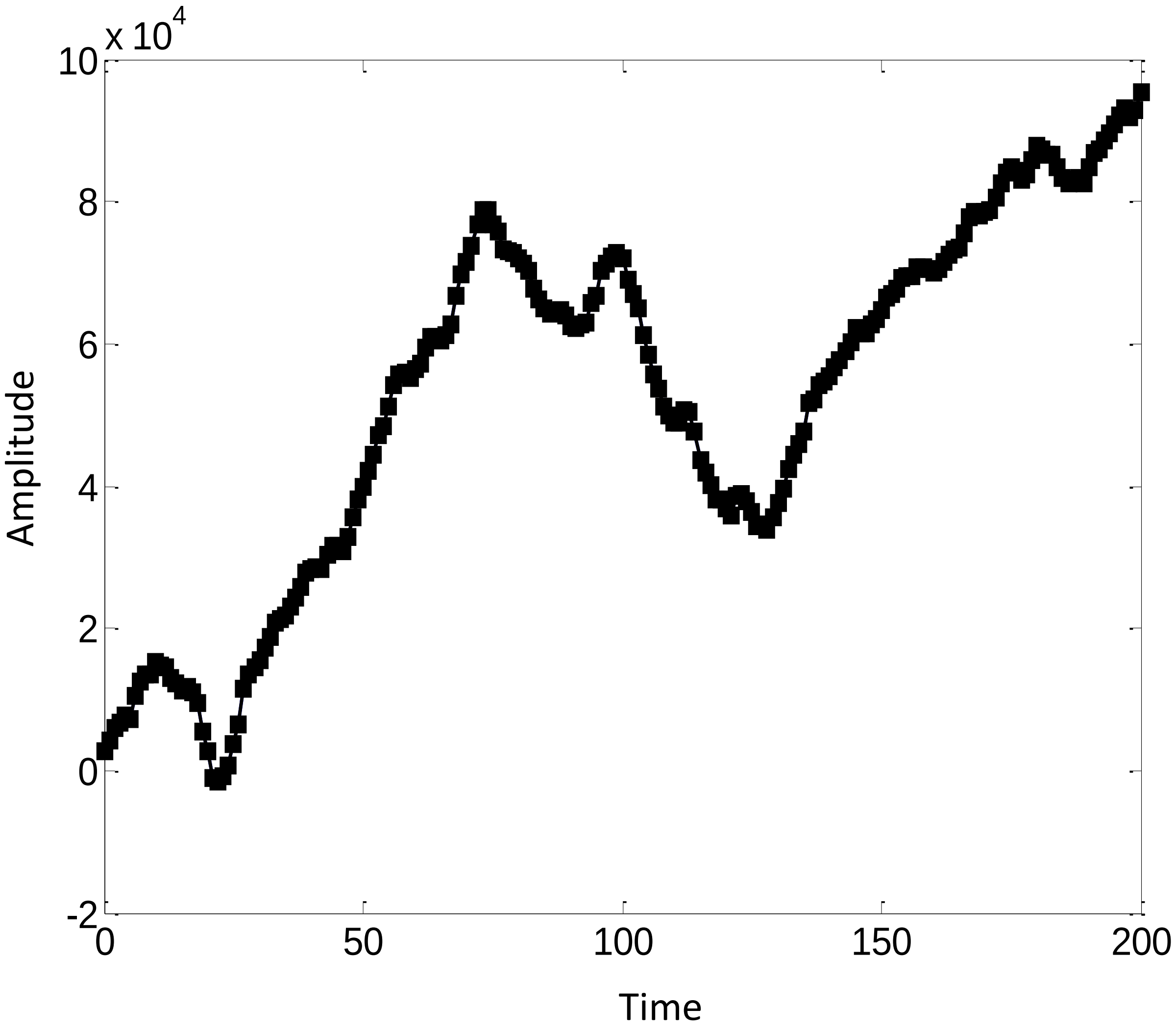
White Phase Modulation and Flicker Phase Modulation



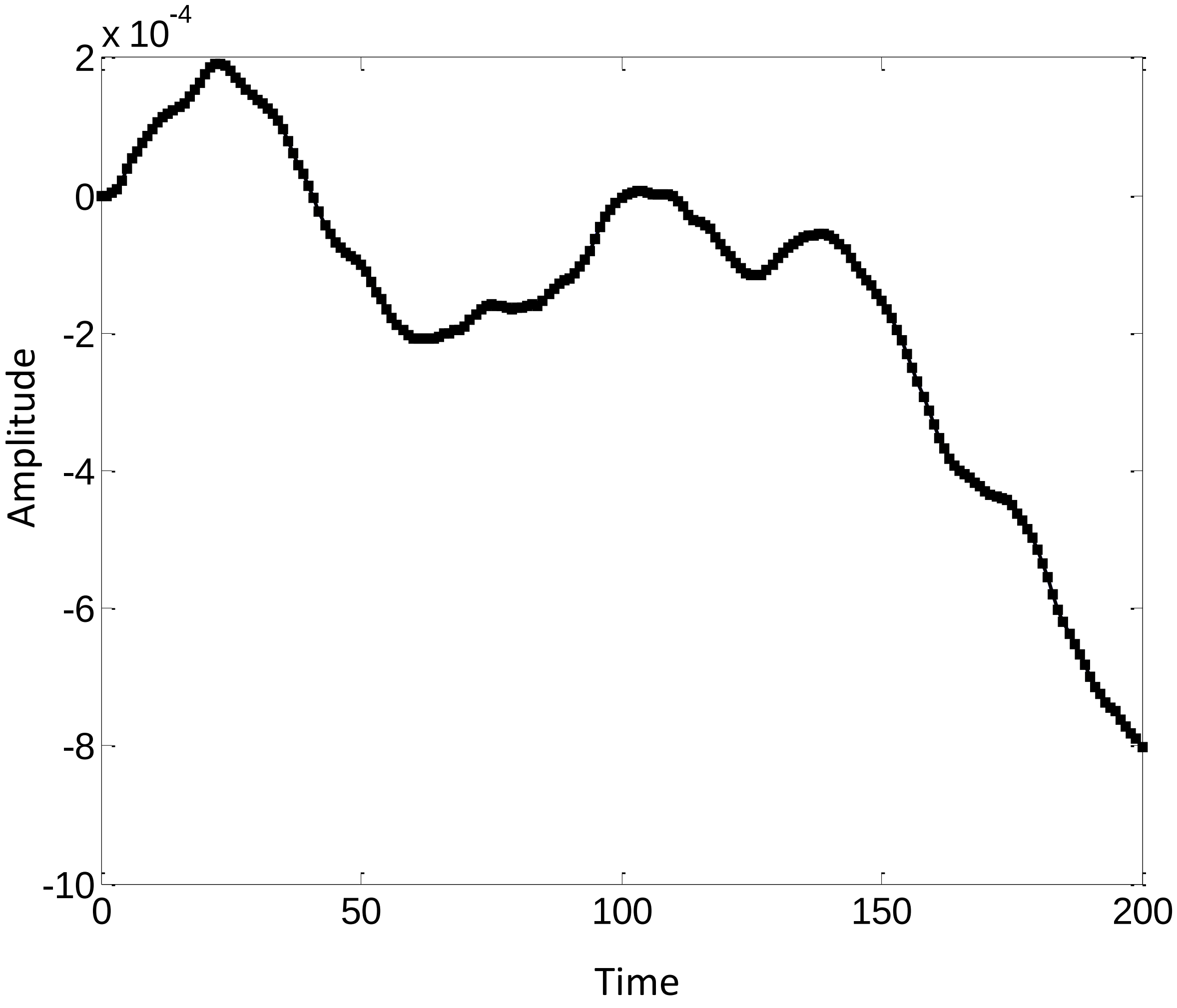


White Frequency Modulation and Flicker Frequency Modulation



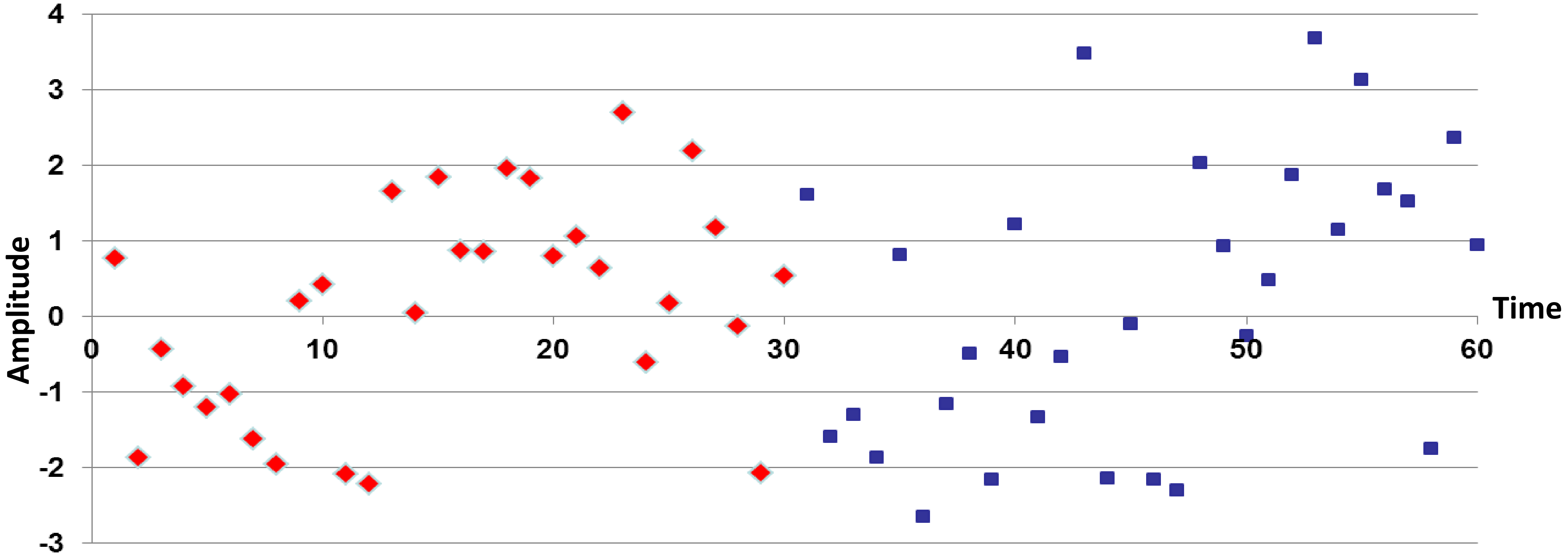


Random Walk Frequency Modulation



Example of a stationary noise process

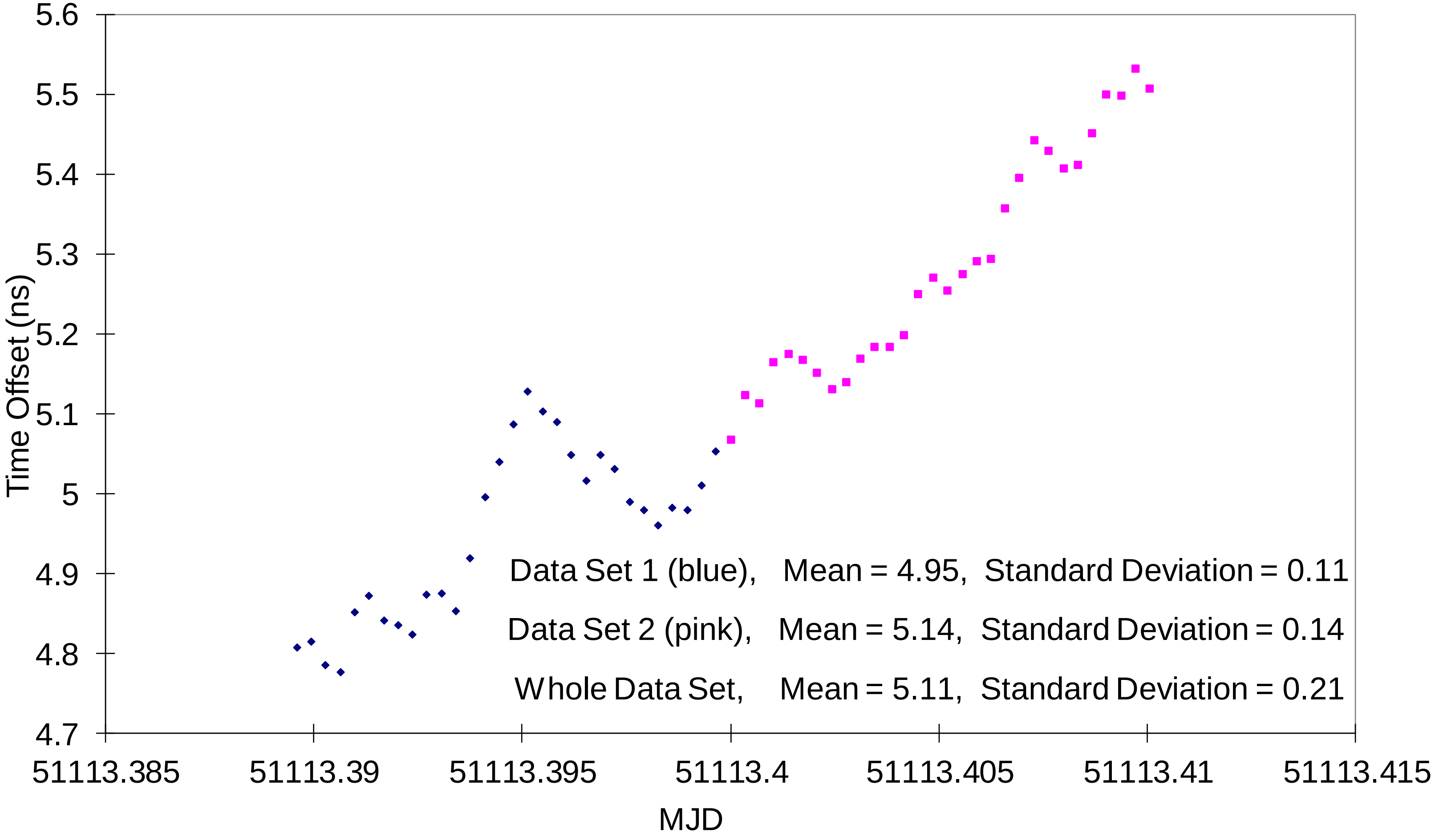
Mean and standard deviation of stationary noise processes are not dependent on length of data set or start point



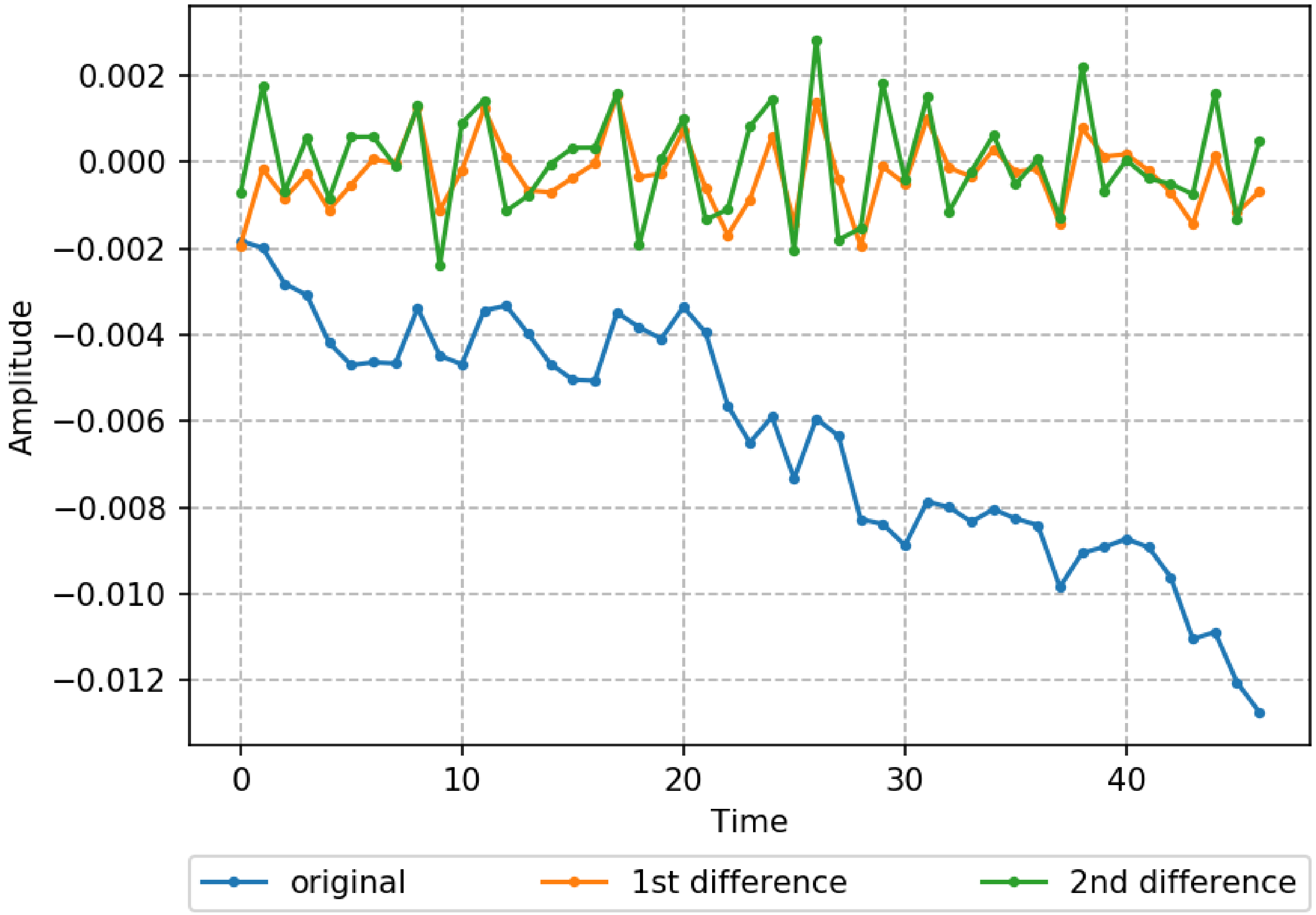
- Mean (red points) = 0.14, standard deviation (red points) = 1.5
- Mean (blue points) = 0.13, standard deviation (blue points) = 1.9
- Mean (all points) = 0.11, standard deviation (sum) = 1.7

Example of a non-stationary noise process

Mean and standard deviation of non-stationary noise processes are dependent on length of data set and start point



Transforming a non-stationary noise process to form a stationary one



1st difference = $x_{i+n} - x_i$

2nd difference = $x_{i+2n} - 2x_{i+n} + x_i$

n = 1 in this example

Second and third difference statistics

Data set is assumed to be complete and evenly spaced

	Equation
AVAR	$\sigma_y^{2}(\tau)=\frac{1}{2(m-2n)n^{2}\tau_0^{2}}\sum_{i=1}^{m-2n}(x_i-2x_{i+n}+x_{i+2n})^{2}$
HVAR	$\sigma_h^{2}(\tau)=\frac{1}{6(m-3n)n^{2}\tau_0^{2}}\sum_{i=1}^{m-3n}(x_i-3x_{i+n}+3x_{i+2n}-x_{i+3n})^{2}$
MVAR	$(\mathrm{Mod}\,\sigma_y)^{2}(\tau)=\frac{1}{2\tau^{2}n^{2}(m-3n+1)}\sum_{j=1}^{m-3n+1}\left[\sum_{i=j}^{n+j-1}(x_{(i+2n)}-2x_{(i+n)}+x_{(i)})\right]^{2}$
TVAR	$\sigma_x^{2}(\tau)=\frac{\tau^{2}}{3}(\mathrm{mod}\,\sigma_y^{2})$

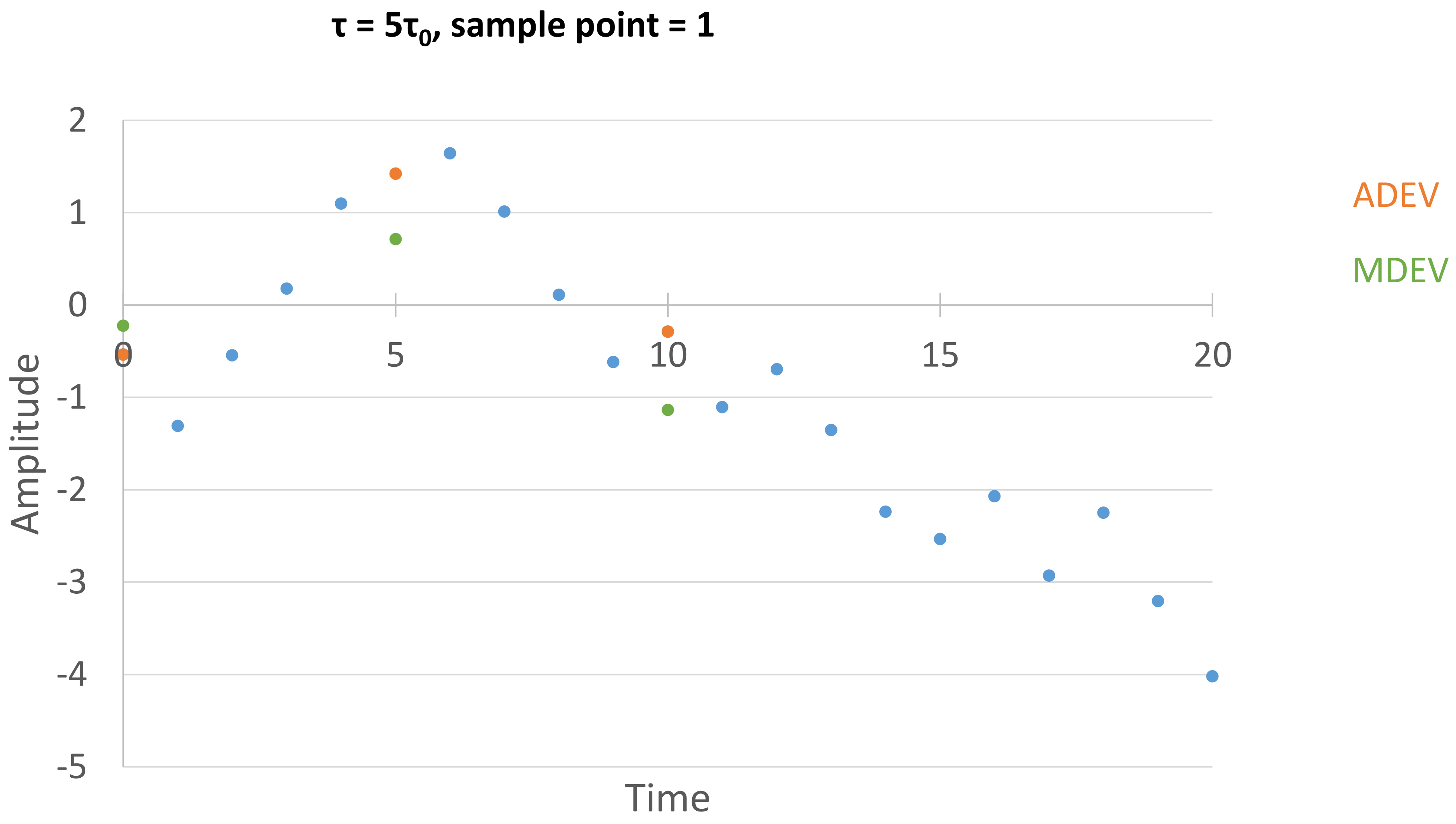
x_i is the phase of the i^{th} measurement
τ_0 is the minimum (normal) spacing of the data set

τ is the averaging time, $\tau = n\tau_0$
m is the total number of data points

Allan Deviation (ADEV)

- May be used with either time or frequency measurements
- Insensitive to time offset, normalised frequency offset but sensitive to linear frequency drift
- Gradient of plots of $\log_{10}$(ADEV) against $\log_{10}(\tau)$ depend on noise type.
- Cannot distinguish between WPM and FPM noise.

Difference between Allan Deviation and Modified Allan Deviation



Modified Allan Deviation

- May be used to distinguish between WPM and FPM noise.
- More computationally intense than using Allan deviation.
- More sensitive to missing data points.
- Useful when large magnitude WPM or FPM is present, may average these out.
- MDEV estimates will have larger uncertainties at a given averaging time compared with ADEV estimates.

Hadamard Deviation and Time Deviation

- HDEV is a third difference statistic
- HDEV and ADEV agree within statistical uncertainty in presence of WFM
- HDEV uncertainties will be greater particularly at large averaging time
- HDEV insensitive to linear frequency drift

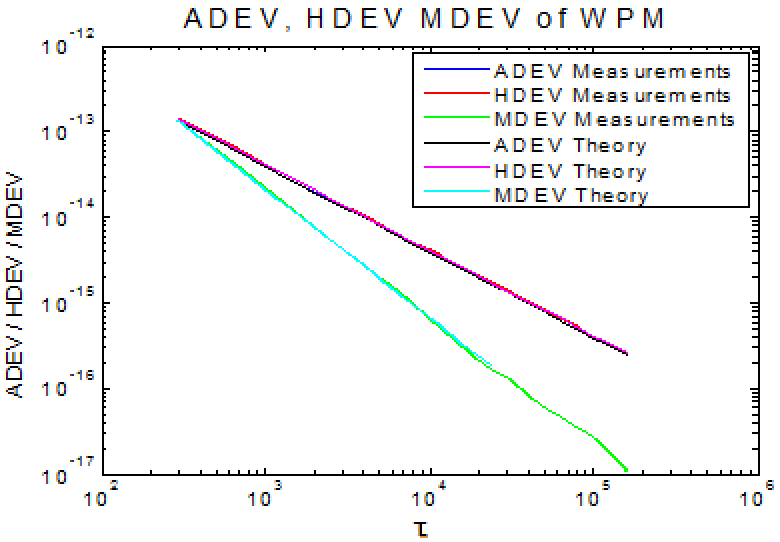
- TDEV used to characterise time offset measurements
- TDEV will have same values as the classical deviation for WPM noise
- Direct relationship to MDEV
- Mainly used for characterising short- term time transfer noise

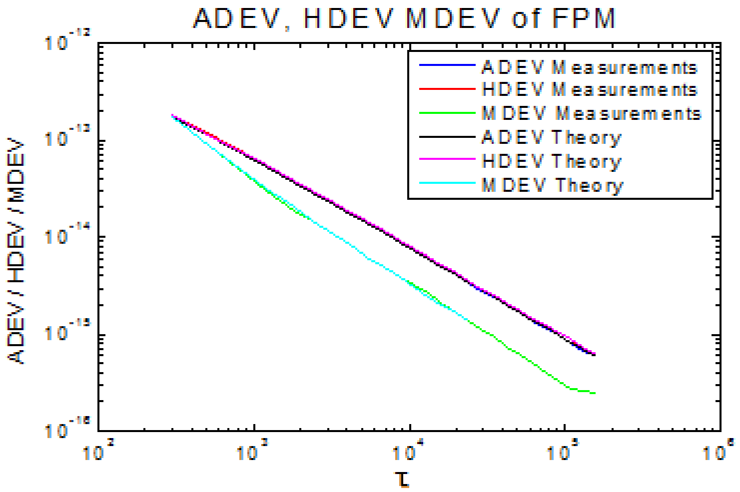
Noise Types

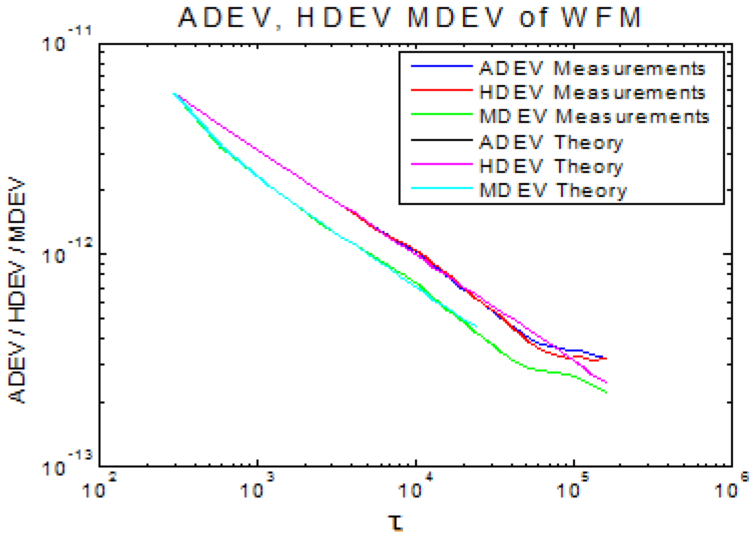
Noise Type	*ADEV (σ_y)* *HDEV*	*MDEV* *(MOD σ_y)*	*TDEV (σ_x)*
White Phase Modulation	-1	-3/2	-1/2
Flicker Phase Modulation	-1	-1	0
White Frequency Modulation	-1/2	-1/2	1/2
Flicker Frequency Modulation	0	0	1
Random Walk Frequency Modulation	1/2	1/2	3/2
Linear Frequency Drift	1	1	2

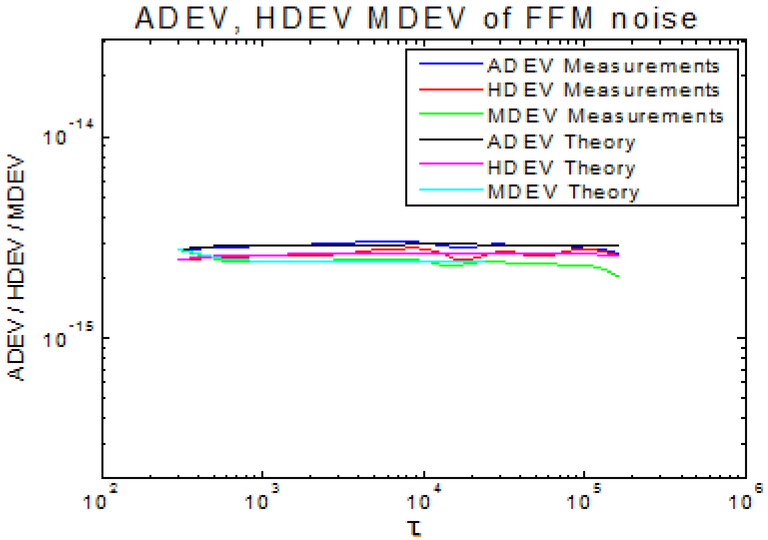
Gradient of plots of $\log_{10}$(ADEV), $\log_{10}$(MDEV) and $\log_{10}$(TDEV) against $\log_{10}(\tau)$ for standard noise types and linear frequency drift

Graphs in the presence of different noise types









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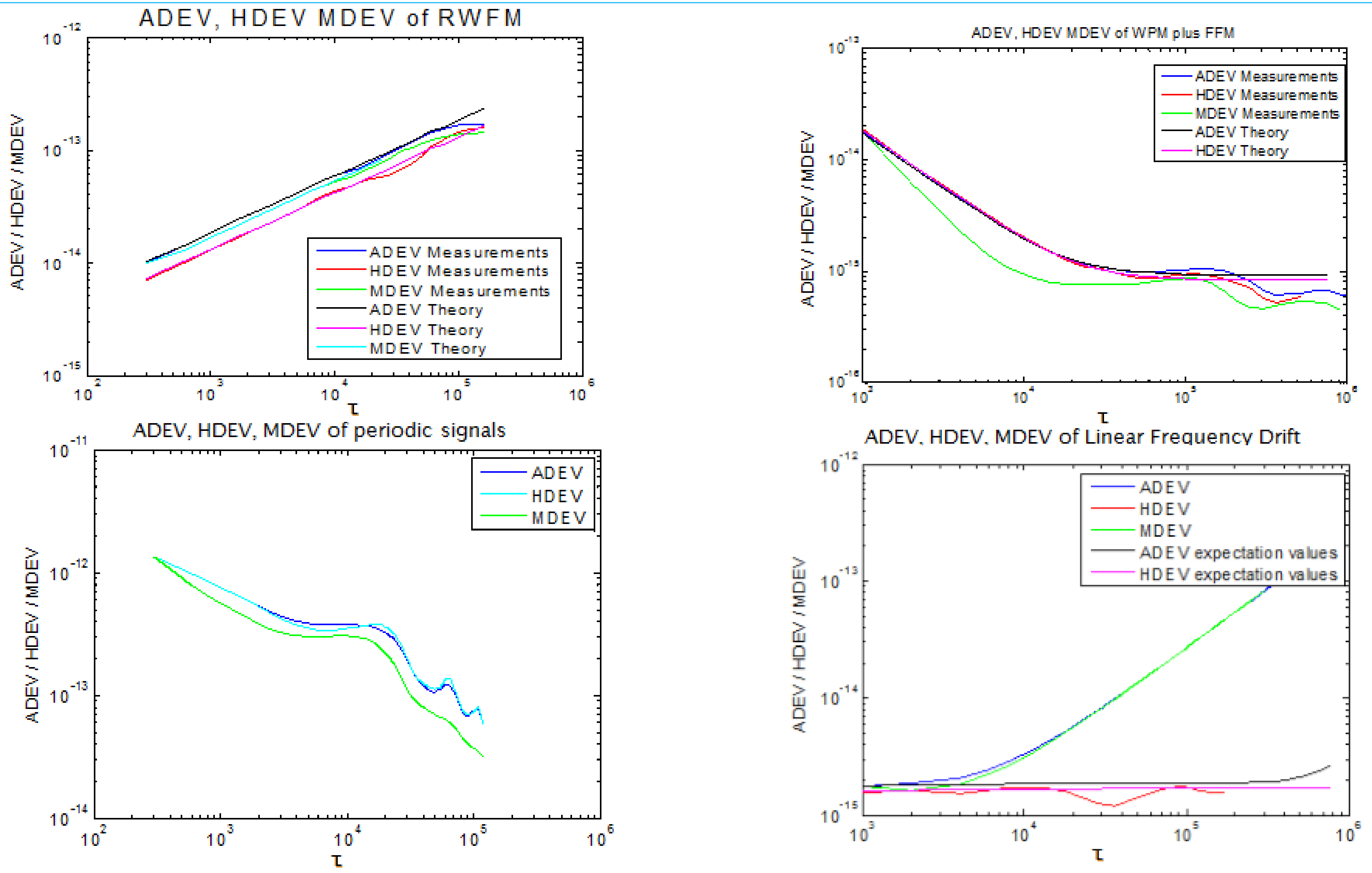


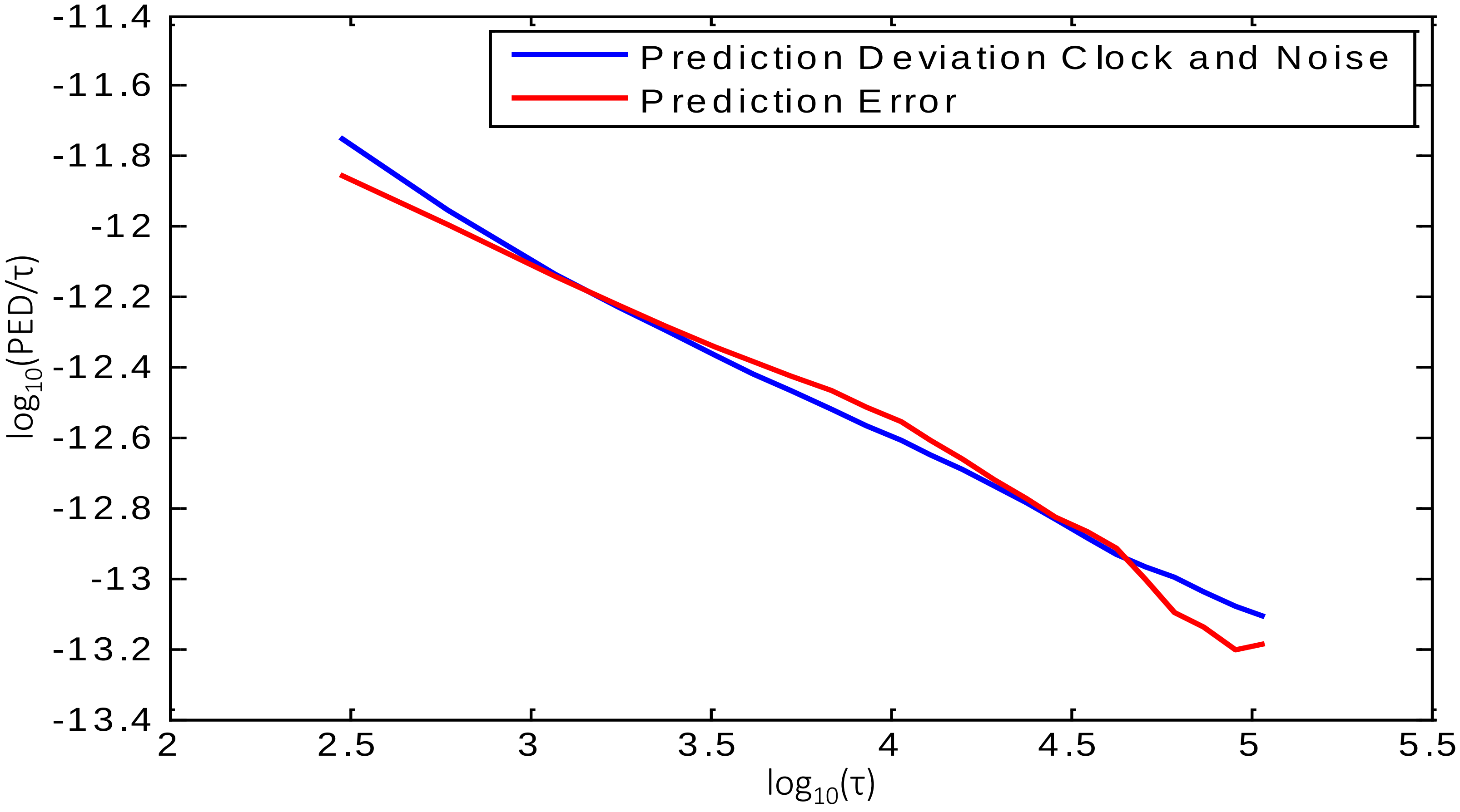
Example: Predicting clock offsets

- Predict time and normalised frequency offset between two clocks or timescale

$$\text{ADEV} = \text{PED}/\tau$$

- Above equation exact in case of WFM and RWFM noise processes, and a reasonable approximation for FFM noise

(PED/τ) prediction of space clocks (real data)



τ is the prediction length

Conclusions

- Short review of atomic clocks
- Explained and demonstrated the usefulness of ADEV, HDEV MDEV and TDEV statistics in characterising clock instabilities.
- Examined how to determine the predictability of an atomic clock from its ADEV statistics.

Thank you for your attention

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CLONETS – CLock NETwork Services

Strategy and innovation for clock services over optical-fibre networks

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- Coordinator
- Participants
- Third Parties